





Please enjoy this complimentary excerpt from Seeing the Math You Teach, Grades K-6.

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## What This Book Is About

This book is intended to help **you** help your children. It was created to provide you with images and descriptions of the way our students learn and think. The images focus on students' understanding of how and why math works, and they are explained in plain and simple terms.

# Why This Matters

The strategies and operations that pop into your adult mind when you think "math" are likely abstract. They may involve symbols and procedures that were "gifted" to you by a caring adult, one who showed you a series of steps that you were then asked to memorize and apply. What we know about math today includes focused attention on developing a deep and sustainable understanding of problems, as well as the perseverance to solve them. Furthermore, understanding and perseverance must be intentionally developed by building on and making connections within the learner's own understanding.

#### **How Mathematics Learning Happens**

Deep understanding in mathematics evolves naturally when students use physical (concrete) materials such as counters, linking cubes, or even cars pulled from a toy box or fruit from a bowl. In this "doing" stage, learners are modeling strategies and operations with physical manipulation.

Similarly, learners can also use visual (pictorial) representations to help them "see" the math. This is when they create visuals that either represent concrete objects such as pictures or sketches, or they construct more abstract representations such as number lines or number bonds to explain their thinking. At first, student-created drawings are littered with detail. For example, when making sense of a problem such as *There were 10 bunnies on the hill, 3 bunnies hopped away* ... the learner may draw complete pictures of bunnies with tails, big feet, and floppy ears. With the right prompting and support, students in this stage can understand that a circle, even one drawn in a tens frame, can represent a bunny. An important part of this mode centers on students making connections between the physical and visual representations.

By making connections between the physical and visual representations, students provide meaning that can also be connected to symbolic understanding, sometimes referred to as the "abstract mode." Here, learners use numbers and other mathematical symbols to model their thinking. A studio artist we know well once said when talking about human anatomy, "People think that they can make abstract art—they think it's easy to just throw paint and shapes on a canvas. A real artist, however, must understand the foundation: the location of the bones, the shape of the muscles. Only when you know how the form looks at its foundation can you make an abstraction of something such as the human form."

In this abstract form, learners can think critically about how they might represent math and their own understanding by basing it on and making connections between the different representations.

Note that as students make connections between and among physical, visual, and symbolic representations, their understanding deepens even further when they talk about their thinking (verbal representations) and connect their thinking to real life (contextual representations). It's the connections between each representation that lead to deeper understanding.

Figure i.1 shows an example of how students might connect representations for place value using base-ten blocks, sketches, and symbols.





Figure i.2 shows an example for how learners might connect representations for equivalent fractions using two-color counters, sketches, and symbols.





Figure i.3 show how students might connect representations for adding integers using algebra tiles, sketches, and symbols.

Figure i.3 Concrete, Representational, and Abstract Representations of Adding Integers



Unfortunately, our school system is often steeped in doing things as they've always been done. This is especially true in math class. While there has been great effort in the last fifteen to twenty years to focus on improving conceptual understanding, there are still many classrooms in which students (and sometimes teachers) often rush to abstract and procedural mathematics, or "just numbers," without spending the time needed to develop deeper understanding with objects, tools, drawings, and the like. When this occurs, math learning is short-circuited as we resort to memorization rather than deep understanding.

Part of your role as a math teacher centers on developing your own understanding of the many ways both to see and represent the math you teach. By knowing and understanding these strategies, you are able to support and encourage students' thinking as they select and create their **own** representations. Only then do students make sense of mathematics; it is the **students** who should be choosing and creating the physical and visual representations, not simply mimicking what the teacher shows them. Please do your best to avoid "show-and-tell teaching"; instead, support students in the ways **they** choose how to show their thinking.

### What the Role of Teacher Looks Like

A skillful teacher is able to probe for understanding by going back and forth between the physical, visual, and symbolic representations, using conversation to help the learner verbally craft connections between their thinking and the way they choose to represent it, and then connect those understandings to contexts in their own lives.

If you are most familiar and accustomed to one or maybe two particular algorithms or strategies for any given operation, this is the perfect opportunity for you to make your own connections. In the pages that follow you will find not only drawings and images of student thinking, but links to videos that model the use of manipulatives. We urge you to ask yourself, "How does this fit with what I know?" "How does this strategy mirror the strategies that I am familiar with using?" "What new ideas does this give me?" We are here to help you answer all of these questions.

# Why These Representations?

It is important to note that we have chosen to model many of the most common strategies developed by learners, those that we see most often in the classroom. Kids, however, are always full of surprises. Some will be likely to invent their own strategies, which may be given equal consideration. Exploration and discussion of student strategies should not be seen as an opportunity for the teacher to **show** students what to "do" and ask that they replicate it. When we acknowledge student thinking and sense-making as a whole group by sharing strategies and asking thoughtful, probing questions, we empower all students.

And that is our most basic intent. We wish to empower you with additional knowledge and tools to enhance your own clarity and understanding in mathematics. We believe you will find what you're looking for here.

#### How to Use This Book

This is not the kind of book you read cover-to-cover. You will most likely use this book as a reference, jumping around as needed to inform the math teaching and learning going on in your classroom. We anticipated you would want to jump around, and we organized the book accordingly.

Let's take a minute to show you how this guide is put together.

There are sixteen color-coded chapters that can be used to quickly locate a topic, and you can jump around to find just what you need to know. Chapters begin with basic topics and go into more detail as you move toward the end. On pages ix–xiii, you'll see a list with all of the different math topics covered in this book. Look for the matching color on the edge of the page to go directly to the chapter.

There is also an additional tool at the back of the book to help you locate specific math problems: the **Topic Index**.

The **Topic Index** lists common math words alphabetically. This more typical way of listing common topics addressed throughout the book provides a handy way to look up topics such as "unit fractions" or "number lines."

Over the years, many teachers have found a variety of ways to use this book in supporting math teaching and learning. Some of those include using the book the following ways:

- As a preparation guide for self-learning about multiple ways to represent mathematical ideas.
- As a planning resource to anticipate a variety of student representations ahead of time.
- During PLC discussions with colleagues.
- To raise family awareness of how children may represent math when doing homework—often by copying a page or two and sending it home ahead of time.
- To support student awareness of representations they may choose from—again, often by copying (or projecting) a page or two and sharing them with students.
- As a reference to the math taught in your curriculum resources or textbooks to provide additional support for representing math thinking, especially in places where the textbook is heavy with "numbers only" work that would benefit from more student visuals.

These are just a few samples of how this book can be used. We encourage you to make it your own, finding new and innovative ways to support your own learning as well as that of your students and their families.

You will note that there are also several videos sprinkled throughout the volume. Since this book primarily centers on visual representations, we wanted to provide support for connecting visuals to physical objects (e.g., manipulatives). Therefore, these videos provide brief snippets for how you might help students connect physical, visual, and symbolic representations. As mentioned before, it's important that you use these to guide your thinking as you design instruction that facilitates students' thinking rather than resorting to "show-and-tell teaching."

We hope you find this to be a valuable resource to validate, reinforce, and expand your thinking and understanding... truly being able to "see" the math you teach so that your students can see it too.