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Please enjoy this complimentary excerpt from *Mastering Math Manipulatives, Grades 4-8*, by Sara Delano Moore and Kimberly Rimbey.

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## Materials

- Virtual two-color counters or physical two-color counters, approximately 20 per student or per pair

## Organization (virtual)

- Getting Started:** Ensure students can access and use the virtual tool. Review annotation tools, the process for taking screenshots, and other important supports.
- Winding Down:** Use screenshots to save student work.

## Mathematical Purpose

In this activity, students use an active addition model to add positive and negative integers. Addition is understood as the action of putting counters into the collection on the desk. Students work with a series of examples to understand the patterns that occur with addends of the same sign or addends of different signs.

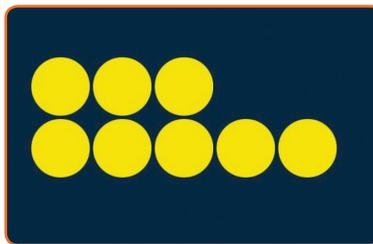
## Manipulative Illustrated

- Didax Two-Color Counters: [www.didax.com/apps/two-color-counters/](http://www.didax.com/apps/two-color-counters/)

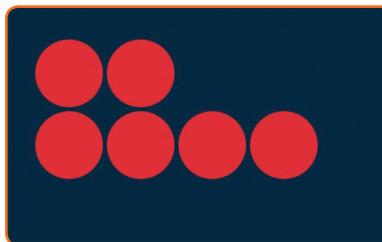
## Steps

- Ask students to represent  $+3$  add  $+5$  (read “positive three add positive five”) with their counters. Support students to share their thinking using the following questions:
  - Where do you see the two addends in your representation?
  - How do we know that the values are positive?
  - How did you act out addition as you built the representation? Where do you see the sum of these values?

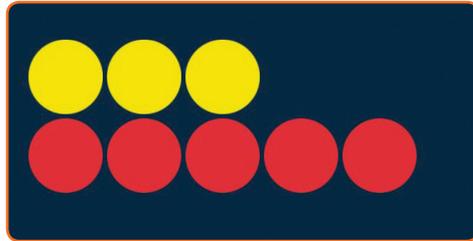
Look for a representation where the two addends are clearly distinguished and all counters are on the positive (non-red) side. Confirm the sum of  $+8$  and ask students to describe how they built their model. Emphasize an active notion of addition, first placing three counters on the positive side and then placing an additional five counters, also on the positive side. Repeat with an additional example or two if necessary.



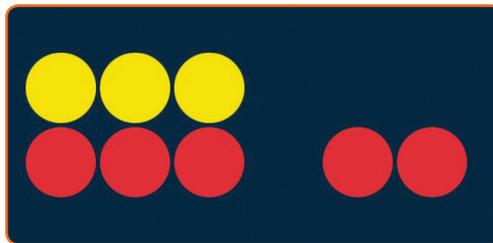
- Ask students to add pairs of negative numbers in the same way. To add  $-2$  plus  $-4$ , for example, first place two counters on the negative side and then place four more counters on the negative side. There are six counters, all negative, so the sum is  $-6$ . Repeat with additional examples, supporting discussion with questions similar to those in Step 1 and encouraging students to see the generalization that the sum of two integers with the same sign is the sum of the values with the sign carried along. The model highlights the fact that there are more of the same piece when adding two numbers of the same sign.



3. Ask students to represent  $+3$  add  $-5$  with their counters. Encourage them to follow the same pattern of placing three counters on the positive (yellow) side and then placing an additional five counters on the negative (red) side. Observe that there are still eight counters on the mat and ask what the sum is. Give students time to discuss their thinking with a partner then lead a class discussion guided by these questions:
- What do you think the sum is? Why?
  - Explain what each counter or group of counters on the mat represents in the problem.
  - Where is  $-2$  on the mat? Why are there more counters there if this is the sum? What do those “extra” counters represent?



4. Support students to understand that a pair of counters, one of each color, has a value of zero. We call these **zero pairs**. Use a number line to help students understand why the pair has a value of zero. Each member of the pair represents a step right or a step left. One step in each direction (a pair of counters) means the location is unchanged in the end. This represents a change of zero. If you have a floor number line in your classroom, have students walk forward and back an equal number of steps to see that they land in the same location where they started.
5. Looking at the problem, there are three zero pairs as each of the three positive counters has a negative counter “partner.” Once the zero pairs are identified, the remaining counters have a value of  $-2$  as there are two red counters left.



6. Repeat this process with a variety of other signed number addends.
7. Once students are comfortable with this, provide pairs of addends with opposite sign positions, such as  $+3$  plus  $-8$  and  $-3$  plus  $+8$ . Encourage students to record their equations in a list so they can notice that both sums have an absolute value of 5 (the difference between 3 and 8) and what the sign is in each case (it matches the addend with the greater absolute value). Finding this pattern, and testing it with other pairs of addends, will help students develop automaticity adding integers and extend their knowledge of basic facts into this realm.

## Why This Manipulative?

This work begins with a quick reminder of first-grade addition, focusing now on the two attributes of signed numbers, the quantity and the sign. This activity is designed to help students focus on the role of each part in adding signed numbers. Two-color counters (or the two-color unit pieces from an algebra tiles or similar set) are particularly helpful because it is easy to see each element, one in quantity and one in color.

The most important work here is the idea of a zero pair. Counters make it easy to see the zero pairs and to move them, as pairs, to a nearby area in the workspace so students can see the net sum clearly. It is important not to sweep the zero pairs back into the main collection of counters so students can check their work and still find the original addends in the model.

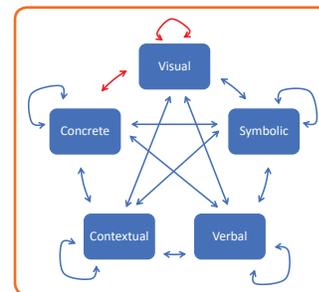
## Developing Understanding

It is important to attend to magnitude in these discussions. While  $-8$  has a greater absolute value than  $-3$ , its value is actually less than  $-3$  because it is located to the left of  $-3$  on the number line. As students generalize their findings about addition with unlike signs, help them be careful in their language.

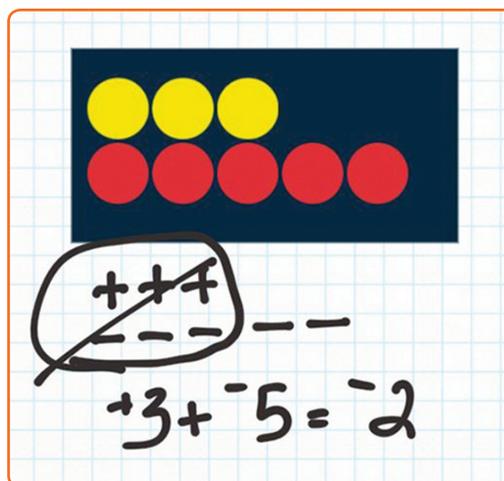
While not all addition situations are active, this approach to addition of integers lays a strong foundation for work with subtraction (in Activity 5.3) and connects with students' intuitive understanding of addition. Once students are confident in their addition skill, provide examples in contexts that include additive comparison (e.g., temperature comparison) or part-part-whole situations (e.g., ionic charge situations) to allow students to use their learning in all contexts where addition is appropriate.

## Featured Connection

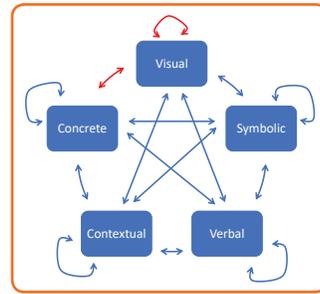
Use the Make a Sketch strategy to connect concrete and pictorial representations of integer addition. When you reinforce students' ability to use a quick sketch to support computation, students are able to find success even when they do not remember a rule or shortcut they might have seen. Encourage students to build their addends in parallel rows or columns. This makes it easy to identify zero pairs with manipulatives or in a sketch. Students can use the  $+$  and  $-$  symbols to represent counters in their sketch.



Source: Lesh, Post, & Behr (1987).



Encourage students to use the Write a Word Problem strategy to identify situations where they will need to add integers. It might be helpful to brainstorm contexts as a class (e.g., altitude, money, ionic charge) before asking students or pairs to write problems for the equations they have solved.



Source: Lesh, Post, & Behr (1987).